



Quintessential inflation from a variable cosmological constant in a 5D vacuum

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Abstract

We explore an effective 4D cosmological model for the universe where the variable cosmological constant governs its evolution and the pressure remains negative along all the expansion. This model is introduced from a 5D vacuum state where the (space-like) extra coordinate is considered as noncompact. The expansion is produced by the inflaton field, which is considered as nonminimally coupled to gravity. We conclude from experimental data that the coupling of the inflaton with gravity should be weak, but variable in different epochs of the evolution of the universe.

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1. Introduction

Recent years have witnessed a large amount of interest in higher-dimensional cosmologies where the extra dimensions are noncompact. A popular example is the so-called Randall–Sundrum brane world (BW) scenario [1]. Particular interest revolves around solutions which are not only Ricci flat, but also Riemann flat ($R^A_{BCD} = 0$), where the vanishing of the Riemann tensor means that we are considering the analog of the Minkowski metric in 5D. A realization of this theory is that all the matter fields in 4D can arise from a higher-dimensional vacuum. One starts with the vacuum Einstein field equations in 5D and dimensional reduction of the Ricci tensor leads to an effective 4D energy–momentum tensor [2]. For this reason the space–time–matter (STM) theory is also called induced-matter (IM) theory. BW and IM theories may appear to be different, but have recently been shown to be equivalent by Ponce de Leon [3].

The potential energy of the scalar field and/or the presence of a variable cosmological term could drive inflation, resolving puzzles such as the monopole, horizon and flatness problems [4]. The variable cosmological term has also been mentioned as a possible solution to the cosmological “constant” problem [5] and, most recently, as a candidate for the dark matter (or quintessence) making up most of the universe [6]. A mechanism for obtaining the decay of the cosmological parameter consists to relax Λ to its present day small value [7–9].

In this Letter we are aimed to study the evolution of the universe which is governed by a variable cosmological constant ($\dot{\Lambda} < 0$) in a 5D vacuum state, such that the expansion of the universe is due to a scalar field (the inflaton field) coupled to gravity. However, on an effective 4D metric the universe evolves with an equation of state with negative pressure p (but with $p \geq -\rho$). This kind of expansion is the well-known quintessential inflation [10].

To describe a scalar field φ , which is nonminimally coupled to gravity in a 5D vacuum state, we consider the metric [11]

$$dS^2 = \psi^2 \frac{\Lambda(t)}{3} dt^2 - \psi^2 e^{2 \int \sqrt{\frac{\Lambda}{3}} dt} dr^2 - d\psi^2, \quad (1)$$

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with the action

$$I = - \int d^4x d\psi \sqrt{\left| \frac{(5)g}{(5)g_0} \right|} \left[\frac{(5)R}{16\pi G} + \frac{1}{2} g^{AB} \varphi_{,A} \varphi_{,B} - \frac{\xi}{2} (5)R \varphi^2 \right]. \quad (2)$$

Here, $\Lambda(t)$ is the decaying cosmological constant ($\dot{\Lambda} < 0$), $G = M_p^{-2}$ is gravitational constant ($M_p = 1.2 \times 10^{19}$ GeV is the Planckian mass), ξ is the coupling of φ with gravity and $(5)R$ is the Ricci scalar, which is zero on the $R_{BCD}^A = 0$ (flat) metric (1). This metric is also 3D spatially isotropic, homogeneous and flat. In such that metric $dr^2 = dx^2 + dy^2 + dz^2$, ψ describes the fifth space-like coordinate and t is the cosmic time. The Lagrange equations give us the equation of motion for φ

$$\ddot{\varphi} + \left[3\sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{\Lambda} \right] \dot{\varphi} - \frac{\Lambda}{3} e^{-2\int \sqrt{\frac{\Lambda}{3}} dt} \nabla_r^2 \varphi - \frac{\Lambda}{3} \left[4\psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] + \xi (5)R \varphi = 0, \quad (3)$$

such that the last term in (3) is zero, because the metric (1) is flat. Furthermore, the commutation expression between φ and $\Pi^t = \frac{\partial \mathcal{L}}{\partial \varphi_{,t}} = \frac{3}{\Lambda \psi^2} \dot{\varphi}$, is

$$\begin{aligned} [\varphi(t, \vec{r}, \psi), \Pi^t(t, \vec{r}', \psi')] \\ = \frac{i}{a_0^3} g^{tt} \left| \frac{(5)g_0}{(5)g} \right| \left(\frac{\Lambda_0}{\Lambda} \right) \delta^{(3)}(\vec{r} - \vec{r}') \delta(\psi - \psi'), \end{aligned} \quad (4)$$

where a_0 is the scale factor of the universe when inflation starts. The field $\varphi(t, \vec{r}, \psi)$ can be transformed as

$$\varphi(t, \vec{r}, \psi) = e^{-\frac{1}{2} \int [3(\frac{\Lambda}{3})^{1/2} - \frac{\dot{\Lambda}}{\Lambda}] dt} \left(\frac{\psi_0}{\psi} \right)^2 \chi(t, \vec{r}), \quad (5)$$

such that, due to the fact $\frac{\partial \varphi}{\partial \psi} = -\frac{2}{\psi} \varphi$, Eq. (3) holds

$$\begin{aligned} \ddot{\varphi} + \left[3\sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{\Lambda} \right] \dot{\varphi} - \frac{\Lambda}{3} e^{-2\int \sqrt{\frac{\Lambda}{3}} dt} \nabla_r^2 \varphi \\ + \left[\frac{2\Lambda}{3} + \xi (5)R \right] \varphi = 0. \end{aligned} \quad (6)$$

Using the transformation (5), we obtain the equation of motion for the field χ

$$\begin{aligned} \ddot{\chi} - \frac{\Lambda}{3} e^{-2\int (\frac{\Lambda}{3})^{1/2} dt} \nabla_r^2 \chi - \left\{ \frac{1}{4} \left[3 \left(\frac{\Lambda}{3} \right)^{1/2} - \frac{\dot{\Lambda}}{\Lambda} \right]^2 \right. \\ \left. + \frac{1}{2} \left[\frac{\dot{\Lambda}}{2} \left(\frac{3}{\Lambda} \right)^{1/2} - \left(\frac{\ddot{\Lambda}}{\Lambda} - \left(\frac{\dot{\Lambda}}{\Lambda} \right)^2 \right) \right] \right. \\ \left. - \left(\frac{2\Lambda}{3} + \xi (5)R \right) \right\} \chi = 0. \end{aligned} \quad (7)$$

The field χ can be written as a Fourier expansion

$$\begin{aligned} \chi(t, \vec{r}, \psi) \\ = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int dk_\psi [a_{k_r k_\psi} e^{i(\vec{k}_r \cdot \vec{r} + k_\psi \psi)} \xi_{k_r k_\psi}(t, \psi) \\ + a_{k_r k_\psi}^\dagger e^{-i(\vec{k}_r \cdot \vec{r} + k_\psi \psi)} \xi_{k_r k_\psi}^*(t, \psi)], \end{aligned} \quad (8)$$

such that

$$[\chi(t, \vec{r}), \dot{\chi}(t, \vec{r}')] = \frac{i}{a_0^3} \delta^{(3)}(\vec{r} - \vec{r}') \delta(\psi - \psi'), \quad (9)$$

and $\xi_{k_r k_\psi}(t, \psi) = e^{-ik_\psi \psi} \bar{\xi}_{k_r k_\psi}(t)$. The commutator (9) is satisfied for $[a_{\vec{k}_r k_\psi}, a_{\vec{k}_r' k_\psi'}^\dagger] = \delta^{(3)}(\vec{k}_r - \vec{k}_r') \delta(k_\psi - k_\psi')$ and $[a_{\vec{k}_r k_\psi}^\dagger, a_{\vec{k}_r' k_\psi'}^\dagger] = [a_{\vec{k}_r k_\psi}, a_{\vec{k}_r' k_\psi'}] = 0$, if the following condition holds:

$$\bar{\xi}_{k_r k_\psi}(t) \dot{\xi}_{k_r k_\psi}^*(t) - \bar{\xi}_{k_r k_\psi}^*(t) \dot{\xi}_{k_r k_\psi}(t) = \frac{i}{a_0^3}. \quad (10)$$

where $\bar{\xi}_{k_r k_\psi}(t)$ satisfies the following equation of motion:

$$\begin{aligned} \ddot{\bar{\xi}}_{k_r k_\psi} + \left\{ \frac{\Lambda}{3} k_r^2 e^{-2\int (\frac{\Lambda}{3})^{1/2} dt} - \left[\frac{1}{4} \left[3 \left(\frac{\Lambda}{3} \right)^{1/2} - \frac{\dot{\Lambda}}{\Lambda} \right]^2 \right. \right. \\ \left. \left. + \frac{1}{2} \left[\frac{\dot{\Lambda}}{2} \left(\frac{3}{\Lambda} \right)^{1/2} - \left(\frac{\ddot{\Lambda}}{\Lambda} - \left(\frac{\dot{\Lambda}}{\Lambda} \right)^2 \right) \right] \right. \right. \\ \left. \left. - \left(\frac{2\Lambda}{3} + \xi (5)R \right) \right] \right\} \bar{\xi}_{k_r k_\psi} = 0. \end{aligned} \quad (11)$$

Hence, the expansion (8) now can be written as

$$\chi(t, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int dk_\psi [a_{k_r k_\psi} e^{i(\vec{k}_r \cdot \vec{r})} \bar{\xi}_{k_r k_\psi}(t) + \text{c.c.}], \quad (12)$$

being c.c. the complex conjugate.

2. Effective 4D dynamics

We consider the metric (1). On the hypersurface $\psi = \sqrt{\frac{3}{\Lambda(t)}}$, the effective 4D metric that results is

$$dS_{\text{eff}}^2 = \left[1 - \frac{3\dot{\Lambda}^2}{4\Lambda^3} \right] dt^2 - \frac{3}{\Lambda} e^{2\int \sqrt{\frac{\Lambda}{3}} dt} dr^2, \quad (13)$$

so that the effective 4D action for this metric is

$$\begin{aligned} {}^{(4)}I = - \int d^4x \sqrt{\left| \frac{(4)g}{(4)g_0} \right|} \left[\frac{(4)R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} \right. \\ \left. - \frac{\xi_{\text{eff}}}{2} (4)R \varphi^2 \right], \end{aligned} \quad (14)$$

where $g_{\mu\nu} = \text{diag}[(1 - \frac{3\dot{\Lambda}^2}{4\Lambda^3}), -a^2(t), -a^2(t), -a^2(t)]$, with $a^2(t) = \frac{3}{\Lambda} e^{2\int (\frac{\Lambda}{3})^{1/2} dt}$ and ξ_{eff} is the effective coupling of φ with gravity on the metric (13). Moreover, $(4)g$ is the determinant of $g_{\mu\nu}$ and $(4)R$ is the Ricci scalar corresponding to the effective 4D metric (13). For this metric, we adopt the comoving frame given by the effective 4D velocities

$$u^t = \sqrt{\frac{4\Lambda^3}{4\Lambda^3 - 3\dot{\Lambda}^2}}, \quad u^r = 0. \quad (15)$$

The condition for the metric (13) to be Lorentzian is $g_{tt} > 0$, which holds when

$$4\Lambda^3 > 3\dot{\Lambda}^2. \quad (16)$$

Hence, in this Letter we shall consider the case for which the condition (16) complies along all the expansion of the universe. The effective 4D Ricci scalar for the metric (13) is

$${}^{(4)}R = -2\Lambda - \frac{24[2\sqrt{3}\Lambda^{11/2}\dot{\Lambda} - 4\Lambda^7 + 2\Lambda^5\ddot{\Lambda} - \sqrt{3}\Lambda^{7/2}\dot{\Lambda}\ddot{\Lambda}]}{[4\Lambda^3 - 3\dot{\Lambda}^2]^2}. \quad (17)$$

From Eqs. (13) and (14), we obtain the equation of motion for $\varphi(t, \vec{r})$

$$\ddot{\varphi} + \left[\frac{\sqrt{|{}^{(4)}g|}}{\sqrt{|{}^{(4)}g|}} + \frac{\dot{g}^{tt}}{g^{tt}} \right] \dot{\varphi} - \frac{\Lambda}{3g^{tt}} e^{-2\int \sqrt{\frac{\Lambda}{3}} dt} \nabla_r^2 \varphi + \xi_{\text{eff}} \frac{{}^{(4)}R}{g^{tt}} \varphi = 0, \quad (18)$$

where

$$g^{tt} = \frac{4\Lambda^3}{4\Lambda^3 - 3\dot{\Lambda}^2}, \quad (19)$$

$$\frac{\sqrt{|{}^{(4)}g|}}{\sqrt{|{}^{(4)}g|}} + \frac{\dot{g}^{tt}}{g^{tt}} = 3\left(\frac{\Lambda}{3}\right)^{1/2} + \frac{3(\ddot{\Lambda}\dot{\Lambda} - 2\dot{\Lambda}\Lambda^2)}{4\Lambda^3 - 3\dot{\Lambda}^2}. \quad (20)$$

Furthermore, we can make the following identification in Eq. (18):

$$V'(\varphi) = \xi_{\text{eff}} \frac{{}^{(4)}R}{g^{tt}} \varphi. \quad (21)$$

From the action (14), the effective 4D scalar potential can be identified as

$$V(\varphi) = \frac{\xi_{\text{eff}} {}^{(4)}R \varphi^2}{2}. \quad (22)$$

The commutation relation between φ and $\Pi^t = \frac{\partial {}^{(4)}\mathcal{L}}{\partial \varphi_{,t}} = g^{tt} \varphi_{,t}$ is

$$[\varphi(t, \vec{r}), \Pi^t(t, \vec{r}')] = \frac{i}{a_0^3} g^{tt} e^{-\int \left[\frac{\sqrt{|{}^{(4)}g|}}{\sqrt{|{}^{(4)}g|}} + \frac{\dot{g}^{tt}}{g^{tt}} \right] dt} \delta(\vec{r} - \vec{r}'). \quad (23)$$

Using the transformation

$$\varphi(t, \vec{r}) = \chi(t, \vec{r}) e^{-\frac{1}{2} \int \left[\frac{\sqrt{|{}^{(4)}g|}}{\sqrt{|{}^{(4)}g|}} + \frac{\dot{g}^{tt}}{g^{tt}} \right] dt}, \quad (24)$$

we obtain

$$\ddot{\chi} - \left(\frac{4\Lambda^3 - 3\dot{\Lambda}^2}{12\Lambda^2} \right) e^{-2\int (\frac{\Lambda}{3})^{1/2} dt} \nabla_r^2 \chi - m^2(t) \chi = 0, \quad (25)$$

where $m^2(t) = f^2(t) - \dot{f}(t) - \xi_{\text{eff}} g_{tt} {}^{(4)}R$, being

$$f(t) = -\frac{3}{2} \left[\left(\frac{\Lambda}{3} \right)^{1/2} + \frac{\dot{\Lambda}(\ddot{\Lambda} - 2\dot{\Lambda}^2)}{(4\Lambda^3 - 3\dot{\Lambda}^2)} \right]. \quad (26)$$

The field χ can be expanded in terms of their modes $a_{k_r} e^{i\vec{k}_r \cdot \vec{r}} \times \eta_{k_r}(t)$, such that the equation of motion of the time-dependent modes $\eta_{k_r}(t)$ are

$$\ddot{\eta}_{k_r} + \left[k_r^2 \left(\frac{4\Lambda^3 - 3\dot{\Lambda}^2}{12\Lambda^2} \right) e^{-2\int (\frac{\Lambda}{3})^{1/2} dt} - m^2(t) \right] \eta_{k_r}(t) = 0. \quad (27)$$

Hence, from Eqs. (23) and (24), we obtain

$$[\chi(t, \vec{r}), \dot{\chi}(t, \vec{r}')] = \frac{i}{a_0^3} \delta(\vec{r} - \vec{r}'). \quad (28)$$

The effective 4D equation of state is $p = \omega_{\text{eff}}(t)\rho$ (p and ρ are respectively the pressure and the energy density), with

$$\omega_{\text{eff}}(t) = -\frac{1}{3} \frac{[24\Lambda^{9/2} + 18\dot{\Lambda}^3/2 - 20\sqrt{3}\Lambda^3\dot{\Lambda} + 15\sqrt{3}\dot{\Lambda}^3 - 24\Lambda^{5/2}\ddot{\Lambda}]}{[8\Lambda^{9/2} + 6\dot{\Lambda}^2\Lambda^{3/2} - 4\sqrt{3}\Lambda^3\dot{\Lambda} + 3\sqrt{3}\dot{\Lambda}^3 - 2\sqrt{3}\dot{\Lambda}^3\Lambda]}. \quad (29)$$

Note that when $\dot{\Lambda} = 0$, one obtains ${}^{(4)}R = 4\Lambda$ and the metric (13) describes exactly an effective FRW metric with a pressure $p_v = -\rho_v = -\frac{\Lambda}{8\pi G}$, in a de Sitter expansion.

3. An example

We consider the case where $\Lambda(t) = 3p^2(t)/t^2$, such that $p(t)$ is given by

$$p(t) = 1.8at^{-n} + \left(\frac{b^2}{4a} - 0.95 \right) + Ct, \quad (30)$$

where $a = \frac{1}{6} 10^{30n} G^{n/2}$, $b = \frac{8}{7} 10^{15n} G^{n/4}$, $C = 2 \times 10^{-61} G^{-1/2}$ and $n = 0.352$. There are at least four significative periods that we can identificate in this model.

- The early period where the equation of state is $p \simeq -\rho$, being $p(t) \gg 1$. In our model this period holds for $t/t_p \ll 10^{10}$ [see Fig. 1]. In this epoch we can make the approximation $\Lambda(t) \simeq \Lambda_0$ (being Λ_0 a constant).
- The period when $p(t) \simeq 1 + \epsilon_1$ ($\epsilon_1 = 0.000184$ in our model), where the equation of state is nearly matter dominated ($p \simeq -\epsilon_2\rho$, being $\omega_{\text{eff}} = \epsilon_2 = -0.00025$ in our model). In our model this epoch holds for $10^{35} < t/t_p < 10^{59}$ [see Fig. 1].
- The period which describes the present day universe, for which $p(t) \simeq 1.898$ and the equation of state is $p = -0.687\rho$. This epoch holds approximately when the universe has an age $t/t_p \simeq 10^{60.652}$.
- The asymptotic evolution of the universe (for our model) where the expansion is a de Sitter expansion with $p(t) = \Lambda_f$, being Λ_f is the asymptotic final value for the cosmological parameter. This epoch holds for $t/t_p > 10^{62}$ [see Fig. 1].

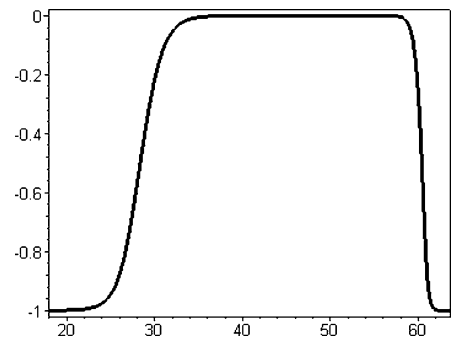


Fig. 1. Evolution of $\omega_{\text{eff}}[x(t)]$ as a function of $x(t) = \log_{10}(t/t_p)$.

In the following subsections we shall study with more detail these different epochs for the evolution of the universe.

3.1. Early (de Sitter) inflationary period: $\Lambda \simeq \Lambda_0$

The early inflationary period in which $p(t) \gg 1$, can be approximated to a nearly de Sitter expansion where $\dot{\Lambda}^2/\Lambda^3 \ll 1$ and hence $\Lambda \simeq \Lambda_0 \simeq 3p^2/t_p^2$. In this epoch, which describes the expansion of the universe for $t \ll 10^{10}t_p$ (in our model), the general solution is given by

$$\eta_{k_r}(t) = A_1 \mathcal{H}_{\nu_1}^{(1)}[y_1(t)] + A_2 \mathcal{H}_{\nu_1}^{(2)}[y_1(t)], \quad (31)$$

such that where $\nu_1 = \sqrt{9 - 48\xi_{\text{eff}}}/2$ and $y_1(t) = k_r e^{-(\frac{\Lambda_0}{3})^{1/2}t}$.

The normalized solution with $A_1 = 0$ and $A_2 = \frac{i}{2}\sqrt{\frac{\pi\Lambda_0}{3}}$, is

$$\eta_{k_r}(t) = \frac{i}{2}\sqrt{\frac{\pi\Lambda_0}{3}} \mathcal{H}_{\nu_1}^{(2)}\left[k_r e^{-\sqrt{\frac{\Lambda_0}{3}}t}\right]. \quad (32)$$

The power of the spectrum for the squared φ -fluctuations calculated on scales $k_r \gg e^{\sqrt{\frac{\Lambda_0}{3}}t}$ (super-Hubble scales), is

$$\langle \varphi^2 \rangle|_{\text{IR}} \sim k_r^{3-2\nu_1}. \quad (33)$$

It implies that this spectrum should be scale invariant only for $\xi_{\text{eff}} = 0$. In particular, we can calculate the range of validity for ξ_{eff} by comparing the spectrum (33) with observational data [12], for the spectral index n_s

$$n_s = 0.97 \pm 0.03. \quad (34)$$

Making, $n_s - 1 = 3 - 2\nu_1$, we obtain

$$-0.008 < \xi_{\text{eff}} < 0. \quad (35)$$

3.2. Matter dominated period: $p = -\epsilon_2\rho$

In our model the equation of state which describes the expansion of the universe for $10^{35} < t/t_p < 10^{59}$ [on the effective 4D metric (13)], is $p/\rho = -\epsilon_2 \simeq -0.00025$, being $\Lambda(t) \simeq 3p^2/t^2$ with $p \simeq (1 + \epsilon_1) = 1.000184$. In this epoch, which describes an (asymptotic) matter dominated universe, the general solution of the equation of motion (27) is given by

$$\eta_{k_r}(t) = C_1 \sqrt{t} \mathcal{H}_{\nu_2}^{(1)}[y_2(t)] + C_2 \sqrt{t} \mathcal{H}_{\nu_2}^{(2)}[y_2(t)], \quad (36)$$

where (C_1, C_2) are constants, $\mathcal{H}_{\nu_2}^{(1,2)}[y_2(t)]$ are the first and second kind Hankel functions and $\nu_2 = \frac{\sqrt{1+4s(p)}}{2p}$, $y_2(t) = k_r \frac{\sqrt{p^2-1}}{p} (\frac{t}{t_0})^{-p}$. The normalized solution is

$$\eta_{k_r}(t) = \frac{i}{2}\sqrt{t}p \sqrt{\frac{\pi}{t_p^3}} \mathcal{H}_{\nu_2}^{(2)}[y_2(t)], \quad (37)$$

where

$$s(p) = \frac{3}{4}(3p^2 + 4p + 1) - 6\xi_{\text{eff}}(2p^2 + 3p + 1). \quad (38)$$

The power of the spectrum for $\langle \varphi^2 \rangle$ on scales $k_r \gg \frac{p}{\sqrt{p^2-1}} \times (t/t_0)^p$, is

$$\langle \varphi^2 \rangle|_{\text{IR}} \sim k_r^{3-2\nu_2}, \quad (39)$$

such that this spectrum is nearly scale invariant for $\xi_{\text{eff}} \simeq 0.111$. From experimental data (34), we obtain

$$0.109 < \xi_{\text{eff}} < 0.111. \quad (40)$$

3.3. Present day epoch

To study the present day epoch, which we estimate as $t_a \simeq 10^{60.652}t_p$, we can approximate $p(t)$ in Eq. (30)

$$p_a(t) \simeq \left(\frac{b^2}{4a} - 0.959\right) + Ct. \quad (41)$$

For $|t-t_a| \ll 1$, $p_a(t)$ can be approximated to a constant, i.e.,

$$p_a(t) \simeq 1.898. \quad (42)$$

Hence, this epoch can be treated as a power-law expanding universe, with [see Eq. (27)]

$$\begin{aligned} \ddot{\eta}_{k_r} + \left[k_r^2 \left(\frac{\Lambda_a(t)}{3} - \frac{0.16\Lambda_a(t)}{4} \right) e^{-2Ct_a} \left(\frac{t}{t_p} \right)^{-2p_a} \right. \\ \left. - m_a^2(t) \right] \eta_{k_r} = 0, \end{aligned} \quad (43)$$

where $\Lambda_a(t) = \frac{3p_a^2}{t^2}$,

$$\begin{aligned} m_a^2(t) &= m^2(p = p_a, t) \\ &= \frac{9}{4} \frac{(p_a + 1)^2}{t^2} - \frac{3}{2} \frac{(p_a + 1)}{t^2} - \frac{6\xi_{\text{eff}}}{t^2} (p_a + 1)(2p_a + 1), \end{aligned} \quad (44)$$

and we have done the approximation $\frac{\dot{\Lambda}^2}{\Lambda^3}|_{t=t_a, p=p_a} \simeq 0.16$. In this epoch $\omega_{\text{eff}} \simeq -0.68$. The normalized solution of Eq. (43), in this quintessential epoch, is

$$\eta_{k_r}(t)|_{|t-t_a| \ll 1} \simeq \frac{i}{2}p_a \sqrt{\frac{\pi t}{t_a^3}} \mathcal{H}_{\nu_3}^{(2)}[y_3(t)], \quad (45)$$

where $\nu_3 = \frac{\sqrt{1+4B}}{2p_a}$, $y_3(t) = k_r \frac{\sqrt{3}}{p_a} (t/t_a)^{-p_a}$, and

$$B = \frac{9}{4}(p_a + 1)^2 - \frac{3}{2}(p_a + 1) - 6\xi_{\text{eff}}(p_a + 1)(2p_a + 1).$$

One can calculate the power of the spectrum for $\langle \varphi^2 \rangle$ on scales $k_r \gg \frac{p_a}{\sqrt{3}}(t/t_a)^{p_a}$, such that

$$\langle \varphi^2 \rangle|_{\text{IR}} \sim k_r^{3-2\nu_3}, \quad (46)$$

which is nearly scale invariant (i.e., $\nu_3 \simeq 3/2$) for $\xi_{\text{eff}} \simeq 0.08$. From observational data for n_s (34), we obtain

$$-0.006 < \xi_{\text{eff}} < 0.08. \quad (47)$$

3.4. Asymptotic de Sitter expansion

In our model the final asymptotic expansion of the universe can be approximated to a nearly de Sitter expansion where $\dot{\Lambda}^2/\Lambda^3 \ll 1$ such that $\Lambda \simeq \Lambda_f \simeq 3C^2$, where we are considering $C = 2 \times 10^{-61} G^{-1/2}$ in the power (30). In this epoch, which describes in our model the expansion of the universe for $t > 10^{62} t_p$, the general solution is

$$\eta_{k_r}(t) = B_1 \mathcal{H}_{\nu_4}^{(1)}[y_4(t)] + B_2 \mathcal{H}_{\nu_4}^{(2)}[y_4(t)], \quad (48)$$

such that $\nu_4 = \sqrt{9 - 48\xi_{\text{eff}}}/2$ and $y_4(t) = k_r e^{-Ct}$.

The normalized solution with $B_1 = 0$ and $B_2 = \frac{i}{2}\sqrt{\pi}C$, is

$$\eta_{k_r}(t) = \frac{i}{2}C\sqrt{\pi}\mathcal{H}_{\nu_4}^{(2)}[k_r e^{-Ct}]. \quad (49)$$

The power of the spectrum for the squared φ -fluctuations on scales $k_r \gg e^{Ct}$, is

$$\langle \varphi^2 \rangle|_{\text{IR}} \sim k_r^{3-2\nu_4}, \quad (50)$$

such that this spectrum become scale invariant for $\xi_{\text{eff}} = 0$. Finally, from (34) we obtain

$$-0.08 < \xi_{\text{eff}} < 0. \quad (51)$$

4. Final comments

In this Letter we have studied a model which describes all the expansion of the universe governed by a decreasing cosmological constant from a 5D vacuum state. When we take a foliation on the fifth (space-like) coordinate $\psi(t) = \sqrt{\frac{\Lambda(t)}{3}}$, the effective 4D dynamics describes an universe which has a 4D equation of state $p = \omega_{\text{eff}}\rho$, with $\omega_{\text{eff}} < 0$. In this model, the expansion of the universe is due to the inflaton field, which is considered as nonminimally coupled to gravity. We have calculated the spectrum for the inflaton field fluctuations on cosmological scales in four different epochs of its evolution.

- In the early inflationary expansion $\omega_{\text{eff}} \simeq -1$ and we obtain that the spectrum of $\langle \varphi^2 \rangle|_{\text{IR}}$ is nearly scale invariant for $-0.08 < \xi_{\text{eff}} < 0$.
- In the matter dominated epoch the universe is well described by $\omega_{\text{eff}} \simeq -0.00025$ and the spectrum of $\langle \varphi^2 \rangle|_{\text{IR}}$ is scale invariant for $0.109 < \xi_{\text{eff}} < 0.111$.
- The third epoch describes the present day universe (which is considered as 1.5×10^{10} years old), with $\omega_{\text{eff}} \simeq -0.68$.

For the spectrum of $\langle \varphi^2 \rangle|_{\text{IR}}$ to be nearly scale invariant we obtain that the coupling must be $-0.06 < \xi_{\text{eff}} < 0.08$.

- The asymptotic universe is described by a cosmological constant with $\Lambda \simeq \Lambda_f = 3C^2$ (being $C = 2 \times 10^{-61} G^{-1/2}$). In this stage (valid for $t \gg 1/C$), the universe evolves as in a 4D de Sitter expansion with $\omega_{\text{eff}} \simeq -1$, such that $\langle \varphi^2 \rangle|_{\text{IR}}$ is nearly scale invariant for $-0.08 < \xi_{\text{eff}} < 0$ (as in the early inflationary expansion).

In view of these results, we conclude that ξ_{eff} cannot be constant along the evolution of the universe. However, ξ_{eff} should be very weak. In particular, in the present day quintessential epoch, the experimental data suggests that the coupling should be nearly zero ($-0.06 < \xi_{\text{eff}} < 0.08$). On the other hand, during the early and future inflationary expansions, observation suggests that ξ_{eff} should be negative, but during the (asymptotic) matter dominated epoch the coupling should be positive.

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